Influence of a medium on pair photoproduction and bremsstrahlung

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Abstract

The creation of electron-positron pair by a photon and the bremsstrahlung of an electron in a medium are considered in high-energy region, where influence of the multiple scattering on the processes (the Landau-Pomeranchuk-Migdal (LPM) effect) becomes essential. The pair photoproduction probability is calculated with an accuracy up to the "next to leading logarithm". The integral characteristics: the radiation length and the total probability of pair photoproduction are analyzed under influence of the LPM effect, and the asymptotic expansions of these characteristics are derived.

1 Introduction

When a charged particle is moving in a medium it scatters on atoms. With probability $\sim \alpha$ this scattering is accompanied by a radiation. At high energy the radiation process occurs over a rather long distance, known as the *formation length* l_c:

$$l_c = \frac{l_0}{1+\gamma^2\vartheta_c^2}, \quad l_0 = \frac{2\varepsilon\varepsilon'}{m^2\omega}, \tag{1.1}$$

where ω is the energy of emitted photon, $\varepsilon(m)$ is the energy (the mass) of a particle, $\varepsilon' = \varepsilon - \omega$, ϑ_c is the characteristic angle of photon emission, the system $\hbar = c = 1$ is used. The spectral distribution of the radiation probability per unit time inside the thick target (the boundary effects are neglected) can be obtained from the general formula for the spectral probability derived in the framework of the operator quasiclassical method (see Eqs.(4.2)-(4.8) in [1]). It can be estimated as

$$dW \sim \frac{\alpha}{\pi l_c}\frac{\Delta^2(l_c)}{m^2+\varepsilon^2\vartheta_c^2}\frac{d\omega}{\omega}, \tag{1.2}$$

where $\alpha = e^2 = 1/137$, $\Delta^2(l_c)$ is the mean square of momentum transfer to a projectile from a medium (or an external field) on the formation length l_c.

If the angle of multiple scattering on the formation length $\vartheta_s \equiv \sqrt{\dot{\vartheta}_s^2 l_c}$ is small comparing with the angle $1/\gamma$ ($\gamma = \varepsilon/m$ is the Lorentz factor), then one can consider scattering as a perturbation and perform the decomposition over "the potential" of a medium. The radiation probability in this case is the incoherent sum of the radiation probabilities on isolated atoms of a medium defined by the Bethe-Heitler formula. One get from (1.2) for the spectral probability of radiation per unit time in the case $\vartheta_s \ll 1/\gamma$ ($\vartheta_c = 1/\gamma,\ \Delta^2 = \varepsilon^2\vartheta_s^2 \ll m^2$)

$$dW \sim \frac{\alpha}{2\pi l_c}\vartheta_s^2\gamma^2\frac{d\omega}{\omega} = \frac{\alpha}{2\pi}\dot{\vartheta}_s^2\gamma^2\frac{d\omega}{\omega}. \tag{1.3}$$

At an ultrahigh energy it is possible that $\vartheta_s \gg 1/\gamma$. In this case the characteristic radiation angle (giving the main contribution into the spectral probability) is defined by the angle of multiple scattering ϑ_s. The self-consistency condition is

$$\vartheta_c^2 = \vartheta_s^2 = \dot{\vartheta}_s^2 l_c \gg \frac{1}{\gamma^2}. \tag{1.4}$$

From the condition (1.4) we find

$$\gamma^2\vartheta_c^4 \simeq \dot{\vartheta}_s^2 l_0, \quad l_c \simeq \frac{l_0}{\gamma^2\vartheta_c^2} \simeq \frac{1}{\gamma}\sqrt{\frac{l_0}{\dot{\vartheta}_s^2}}. \tag{1.5}$$

In this case one get from (1.2) for the estimate of the spectral radiation probability per unit time

$$dW \sim \frac{\alpha}{\pi l_c}\frac{d\omega}{\omega} = \frac{\alpha}{\pi}\frac{\dot{\vartheta}_s^2\gamma^2}{\nu_0}\frac{d\omega}{\omega},$$

$$\nu_0^2 = \dot{\vartheta}_s^2 \gamma^2 l_0 = \frac{4\pi Z^2 \alpha^2 n_a}{m^2} L l_0 \gg 1, \quad L = \ln\left[\frac{a_s^2}{\lambda_c^2}\left(1+\gamma^2\vartheta_c^2\right)\right], \qquad (1.6)$$

where Z is the charge of the nucleus, n_a is the number density of atoms in the medium, $\lambda_c = 1/m = (\hbar/mc)$ is the electron Compton wavelength, a_s is the screening radius of the atom.

So, the formula (1.2) gives the general description of the radiation process in terms of the mean momentum transfer valid both in a medium and in an external field, while formulas (1.3) and (1.6) describe the process probability in the particular regimes in a medium.

Landau and Pomeranchuk were the first who showed that if the formation length of bremsstrahlung becomes comparable to the distance over which the multiple scattering becomes important, the bremsstrahlung will be suppressed [2]. Migdal [3] developed the quantitative theory of this phenomenon.

New activity with the theory of the LPM effect (see [4], [5], [6]) is connected with a very successful series of experiments performed at SLAC recently (see [7], [8]). In these experiments the cross section of the bremsstrahlung of soft photons with energy from 200 keV to 500 MeV from electrons with energy 8 GeV and 25 GeV is measured with an accuracy of the order of a few percent. Both LPM and dielectric suppression are observed and investigated. These experiments were the challenge for the theory since in all the mentioned papers calculations are performed to logarithmic accuracy which is not enough for description of the new experiment. The contribution of the Coulomb corrections (at least for heavy elements) is larger then experimental errors and these corrections should be taken into account.

Authors developed the new approach to the theory of the Landau-Pomeranchuk-Migdal (LPM) effect [9] in which the cross section of the bremsstrahlung process in the photon energies region where the influence of the LPM is very strong was calculated with a term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. In the photon energy region, where the LPM effect is "turned off", the obtained cross section gives the exact Bethe-Heitler cross section (within power accuracy) with the Coulomb corrections. This important feature was absent in the previous calculations. Some important features of the LPM effect were considered also in [10], [11], [12].

The crossing process for the bremsstrahlung is the pair creation by a photon. The created particles undergo here the multiple scattering. It should be emphasize that for the bremsstrahlung the formation length (1.4) increases strongly if $\omega \ll \varepsilon$. Just because of this the LPM effect was investigated at SLAC at a relatively low energy. For the pair creation the formation length $l_p = \dfrac{2\varepsilon(\omega-\varepsilon)}{m^2\omega}$ attains maximum at $\varepsilon = \omega/2$ and this maximum is $l_{p,max} = (\omega/2m)\lambda_c$. Because of this even for heavy elements the effect of multiple scattering becomes noticeable starting from $\omega \sim 10$ TeV. Nevertheless it is evident that one have to take into account the influence of a medium on the pair creation and on the

bremsstrahlung hard part of the spectrum in electromagnetic showers being created by the cosmic ray particles of the ultrahigh energies. These effects can be quite significant in the electromagnetic calorimeters operating in the detectors on the colliders in TeV range.

In the present paper both the spectral probability and the integral probability of the pair creation are calculated within an accuracy up to "the next to logarithm" and with the Coulomb correction taken into account (Sec.2). In Sec.3 the radiation length is calculated under influence of the LPM effect. The total probability of photon radiation is considered also. In the Appendixes the technical details of calculation are given.

2 Influence of multiple scattering on pair creation process

The probability of the pair creation by a photon can be obtained from the probability of the bremsstrahlung with help of the substitution law:

$$\omega^2 d\omega \rightarrow \varepsilon^2 d\varepsilon, \quad \omega \rightarrow -\omega, \quad \varepsilon \rightarrow -\varepsilon, \tag{2.1}$$

where ω is the photon energy, ε is the energy of the particle. Making this substitution in Eq.(2.12) of [9] we obtain the spectral distribution of the pair creation probability (over the energy of the created electron)

$$\frac{dW_p}{d\varepsilon} = \frac{2\alpha m^2}{\varepsilon\varepsilon'} \mathrm{Im} \left\langle 0|s_1 \left(G^{-1} - G_0^{-1}\right) + s_2 \mathbf{p} \left(G^{-1} - G_0^{-1}\right) \mathbf{p}|0\right\rangle, \tag{2.2}$$

where

$$\begin{aligned}
& s_1 = 1, \quad s_2 = \frac{\varepsilon^2 + \varepsilon'^2}{\omega^2}, \quad \varepsilon' = \omega - \varepsilon; \\
& G_0 = \mathcal{H}_0 + 1, \quad \mathcal{H}_0 = \mathbf{p}^2, \quad \mathbf{p} = -i\boldsymbol{\nabla}_{\boldsymbol{\varrho}}, \quad G = \mathcal{H} + 1, \quad \mathcal{H} = \mathbf{p}^2 - iV(\boldsymbol{\varrho}), \\
& V(\boldsymbol{\varrho}) = Q\boldsymbol{\varrho}^2 \left(L_1 + \ln\frac{4}{\boldsymbol{\varrho}^2} - 2C\right), \quad Q = \frac{2\pi Z^2\alpha^2\varepsilon\varepsilon' n_a}{m^4\omega}, \quad L_1 = \ln\frac{a_{s2}^2}{\lambda_c^2}, \\
& \frac{a_{s2}}{\lambda_c} = 183 Z^{-1/3} \mathrm{e}^{-f}, \quad f = f(Z\alpha) = (Z\alpha)^2 \sum_{k=1}^{\infty} \frac{1}{k(k^2 + (Z\alpha)^2)},
\end{aligned} \tag{2.3}$$

where $C = 0.577216\dots$ is Euler's constant, n_a is the number density of atoms in the medium, $\boldsymbol{\varrho}$ is the coordinate in the two-dimensional space measured in the Compton wavelength λ_c, which is conjugate to the space of the transverse momentum transfers measured in the electron mass m. The mean value in (2.2) is taken over the states with the definite value of the operator $\boldsymbol{\varrho}$ (see [9], Sec.2). The contribution of scattering of the created electron and positron on the atomic electrons can be incorporated into the effective potential $V(\boldsymbol{\varrho})$ by substitution

$$Q \rightarrow Q_{ef} = Q(1 + \frac{1}{Z}), \quad a_{s2} \rightarrow a_{ef} = a_{s2} \exp\left[\frac{1.88 + f(Z\alpha)}{1 + Z}\right]. \tag{2.4}$$

The potential $V(\boldsymbol{\varrho})$ in Eq.(2.3) we write in the form

$$
\begin{aligned}
&V(\boldsymbol{\varrho}) = V_c(\boldsymbol{\varrho}) + v(\boldsymbol{\varrho}), \quad V_c(\boldsymbol{\varrho}) = q\boldsymbol{\varrho}^2, \quad q = QL_c, \\
&L_c \equiv L(\varrho_c) = \ln\frac{a_{s2}^2}{\lambda_c^2\varrho_c^2}, \quad v(\boldsymbol{\varrho}) = -\frac{q\boldsymbol{\varrho}^2}{L_c}\left(\ln\frac{\boldsymbol{\varrho}^2}{4\varrho_c^2} + 2C\right),
\end{aligned}
\tag{2.5}
$$

where the parameter ϱ_c is defined by the set of equations:

$$
\varrho_c = 1 \quad \text{for} \quad 4QL_1 \leq 1; \quad 4Q\varrho_c^4\ln\frac{a_{s2}^2}{\lambda_c^2\varrho_c^2} = 1 \quad \text{for} \quad 4QL_1 \geq 1, \tag{2.6}
$$

where L_1 is defined in Eq.(2.3). The parameter $\varrho_c \simeq 1/p_c$ is determined by the characteristic angles of created particles with respect to the initial photon momentum (or the corresponding momentum transfers). In accordance with such division of the potential we present the propagators in the expression (2.2) as

$$
G^{-1} - G_0^{-1} = G^{-1} - G_c^{-1} + G_c^{-1} - G_0^{-1} \tag{2.7}
$$

where

$$
G_c = \mathcal{H}_c + 1, \quad G = \mathcal{H}_c + 1 - iv, \quad \mathcal{H}_c = \mathbf{p}^2 - iq\boldsymbol{\varrho}^2
$$

This representation of the propagator G^{-1} permits one to expand it over the "perturbation" v. Indeed, with an increase of q the relative value of the perturbation diminishes ($\dfrac{v}{V_c} \sim \dfrac{1}{L_c}$) since the effective impact parameter diminishes and, correspondingly, the value of logarithm L_c in (2.5) increases. The maximal value of L_c is determined by a size of a nucleus R_n

$$
L_{max} = \ln\frac{a_s^2}{R_n^2} \simeq 2L_1, \tag{2.8}
$$

where $a_s = a_{s2}\exp(f - 1/2) = 111Z^{-1/3}\lambda_c$. When $\varrho_c \ll R_n$ one cannot consider the potential of a nucleus as the potential of a point charge. In this case the expression for the potential $V(\boldsymbol{\varrho})$ has been obtained in [9], Appendix B

$$
V(\boldsymbol{\varrho}) = q\boldsymbol{\varrho}^2(L_{max} - 0.0407).
$$

The matrix elements of the operator G_c^{-1} was calculated explicitly in [9]:

$$
\begin{aligned}
&< \boldsymbol{\varrho}_1|G_c^{-1}|\boldsymbol{\varrho}_2 >= i\int_0^\infty dt e^{-it} < \boldsymbol{\varrho}_1|\exp(-i\mathcal{H}_c t)|\boldsymbol{\varrho}_2 >, \\
&< \boldsymbol{\varrho}_1|\exp(-i\mathcal{H}_c t)|\boldsymbol{\varrho}_2 > \equiv K_c(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t) \\
&= \frac{\nu}{4\pi i\sinh\nu t}\exp\left\{\frac{i\nu}{4}\left[(\boldsymbol{\varrho}_1^2 + \boldsymbol{\varrho}_2^2)\coth\nu t - \frac{2}{\sinh\nu t}\boldsymbol{\varrho}_1\boldsymbol{\varrho}_2\right]\right\},
\end{aligned}
\tag{2.9}
$$

where $\nu = 2\sqrt{iq}$. Substituting this expression in the formula for the spectral distribution of the pair creation probability (2.2) we have

$$\frac{dW_p^c}{d\varepsilon} = \frac{\alpha m^2}{2\pi\varepsilon\varepsilon'}\mathrm{Im}\ \Phi_p(\nu),$$

$$\Phi_p(\nu) = \nu \int\limits_0^\infty dt e^{-it}\left[s_1\left(\frac{1}{\sinh z} - \frac{1}{z}\right) - i\nu s_2\left(\frac{1}{\sinh^2 z} - \frac{1}{z^2}\right)\right]$$

$$= s_1\left(\ln p - \psi\left(p + \frac{1}{2}\right)\right) + s_2\left(\psi(p) - \ln p + \frac{1}{2p}\right), \qquad (2.10)$$

where $z = \nu t$, $p = i/(2\nu)$, $\psi(x)$ is the logarithmic derivative of the gamma function. Some details of the derivation of the last line can be found in Appendix A (see (A.1)-(A.8)). This formula gives the spectral distribution of the pair creation probability in the logarithmic approximation which was used also by Migdal [3]. It should be noted that the parameter ϱ_c entering into the parameter ν (see Eqs.(2.3) and (2.5)) is defined up to the factor ~ 1, what is inherent in the logarithmic approximation. However, below we will calculate the next term of the decomposition over $v(\boldsymbol{\varrho})$ (an accuracy up to the "next to leading logarithm") and this permits to obtain the result which is independent of the parameter ϱ_c. It will be shown that the definition of the parameter ϱ_c in Eq.(2.6) minimizes corrections to (2.10) practically for all values of the parameter ϱ_c. It should be emphasized also that here the Coulomb corrections are included into the parameter ν in contrast to [3].

Let us expand the expression $G^{-1} - G_c^{-1}$ over powers of v

$$G^{-1} - G_c^{-1} = G_c^{-1}(iv)G_c^{-1} + G_c^{-1}(iv)G_c^{-1}(iv)G_c^{-1} + ... \qquad (2.11)$$

Substituting this expansion in (2.6) and then in (2.2) we obtain the decomposition of the probability of the pair creation. Let us note that for $q \ll 1$ the sum of the probability of the pair creation $\dfrac{dW_p^c}{d\varepsilon}$ (2.10) and the first term of the expansion (2.11) gives the Bethe-Heitler spectrum of electron of created pair, see below (2.22). At $q \geq 1$ the expansion (2.11) is the series over powers of $\dfrac{1}{L_c}$. It is important that the variation of the parameter ϱ_c by a factor order of 1 has an influence on the dropped terms in (2.11) only.

In accordance with (2.7) and (2.11) we present the probability of radiation in the form

$$\frac{dW_p}{d\varepsilon} = \frac{dW_p^c}{d\varepsilon} + \frac{dW_p^1}{d\varepsilon} + \frac{dW_p^2}{d\varepsilon} + ... \qquad (2.12)$$

The probability of pair creation $\dfrac{dW_p^c}{d\varepsilon}$ is defined by Eq.(2.10). In formula (2.2) with allowance for (2.7) there is the expression

$$< 0|G^{-1} - G_c^{-1}|0 >= -i\int\limits_0^\infty dt_1 \int\limits_0^\infty dt_2 e^{-i(t_1+t_2)} \int d^2\varrho K_c(0, \boldsymbol{\varrho}, t_1)v(\boldsymbol{\varrho})K_c(\boldsymbol{\varrho}, 0, t_2)$$

$$
+i\int_0^\infty dt_1\int_0^\infty dt_2\int_0^\infty dt_3 e^{-i(t_1+t_2+t_3)}\int d^2\varrho_1\int d^2\varrho_2 K_c(0,\boldsymbol{\varrho}_1,t_1)v(\boldsymbol{\varrho}_1)K_c(\boldsymbol{\varrho}_1,\boldsymbol{\varrho}_2,t_2)
$$
$$
\times v(\boldsymbol{\varrho}_2)K_c(\boldsymbol{\varrho}_2,0,t_3)+..., \tag{2.13}
$$

where the matrix element K_c is defined by (2.9). The term $\dfrac{dW_p^1}{d\varepsilon}$ in (2.12) corresponds to the first term (linear in v) in (2.13). Substituting (2.9) we have

$$
\frac{dW_p^1}{d\varepsilon}=-\frac{2\alpha m^2}{\varepsilon\varepsilon'}\mathrm{Re}\int_0^\infty dt_1\int_0^\infty dt_2 e^{-i(t_1+t_2)}\int d^2\varrho v(\boldsymbol{\varrho})\frac{q^2}{\pi^2\nu^2}\frac{1}{\sinh\nu t_1}\frac{1}{\sinh\nu t_2}
$$
$$
\times\exp\left[-\frac{q\varrho^2}{\nu}\left(\coth\nu t_1+\coth\nu t_2\right)\right]\left[s_1+\frac{4q^2\varrho^2}{\nu^2\sinh\nu t_1\sinh\nu t_2}s_2\right]. \tag{2.14}
$$

Substituting in (2.14) the explicit expression for $v(\boldsymbol{\varrho})$ and integrating over $d^2\varrho$ and $d(t_1-t_2)$ we obtain the following formula for the first correction to the pair creation probability

$$
\frac{dW_p^1}{d\varepsilon}=-\frac{\alpha m^2}{4\pi\varepsilon\varepsilon' L}\mathrm{Im}\ F_p(\nu);\quad F_p(\nu)=\int_0^\infty\frac{dze^{-it}}{\sinh^2 z}\left[s_1f_1(z)-2is_2f_2(z)\right],
$$
$$
f_1(z)=\left(\ln\varrho_c^2+\ln\frac{\nu}{i}-\ln\sinh z-C\right)g(z)-2\cosh zG(z),
$$
$$
f_2(z)=\frac{\nu}{\sinh z}\left(f_1(z)-\frac{g(z)}{2}\right),
$$
$$
g(z)=z\cosh z-\sinh z,\quad t=t_1+t_2,\ z=\nu t
$$
$$
G(z)=\int_0^z(1-y\coth y)dy
$$
$$
=z-\frac{z^2}{2}-\frac{\pi^2}{12}-z\ln\left(1-e^{-2z}\right)+\frac{1}{2}\mathrm{Li}_2\left(e^{-2z}\right), \tag{2.15}
$$

here $\mathrm{Li}_2(x)$ is the Euler dilogarithm. Use of the last representation of function $G(z)$ simplifies the numerical calculation.

As it was said above (see (2.6), (2.8)), $\varrho_c=1$ at

$$
|\nu^2|=\nu_1^2=4QL_1\leq 1\quad (q=QL_1). \tag{2.16}
$$

If the parameter $|\nu|>1$, the value of ϱ_c is defined from the equation (2.6). Then one has

$$
\ln\varrho_c^2+\ln\frac{\nu}{i}=\frac{1}{2}\ln(\varrho_c^4 4QL_c)-i\frac{\pi}{4}=-i\frac{\pi}{4},\quad \varrho_c^4 4QL_c=1. \tag{2.17}
$$

So, we have that the factor at $g(z)$ in the expression for $f_1(z)$ in (2.15) can be written in the form

$$
(\ln\varrho_c^2+\ln\frac{\nu}{i}-\ln\sinh z-C)\rightarrow(\ln\nu_1\vartheta(1-\nu_1)+\ln(\nu_0\varrho_c^2)\vartheta(\nu_1-1)
$$
$$
-i\frac{\pi}{4}-\ln\sinh z-C)=\ln\nu_1\vartheta(1-\nu_1)-i\frac{\pi}{4}-\ln\sinh z-C \tag{2.18}
$$

where

$$\nu_0^2 \equiv |\nu|^2 = 4q = 4QL(\varrho_c) = \frac{8\pi n_a Z^2\alpha^2\varepsilon\varepsilon'}{m^4\omega}L(\varrho_c), \tag{2.19}$$

$\vartheta(x)$ is the Heaviside step function. So, we have two representation of $|\nu|$ depending on ϱ_c: at $\varrho_c = 1$ it is $|\nu| = \nu_1$ and at $\varrho_c \leq 1$ it is $|\nu| = \nu_0$.

When the scattering of created particles is weak ($\nu_1 \ll 1$), the main contribution in (2.15) gives the region where $z \ll 1$. Then

$$\begin{aligned} f_1(z) &\simeq -(C + \ln(it))\frac{z^3}{3} + \frac{2}{9}z^3 = \frac{z^3}{3}(\frac{2}{3} - C - \ln(it)), \\ F_p(\nu) &= -\frac{1}{9}\nu^2\left(s_2 - s_1\right), \quad L \to L_1. \end{aligned} \tag{2.20}$$

Substituting the expansion (C.1) into Eq.(2.10) we find the corresponding asymptotic decomposition of the function $\Phi_p(\nu)$

$$\Phi_p(\nu) \simeq s_1\left(\frac{\nu^2}{6} + \frac{7\nu^4}{60} + \frac{31\nu^6}{126}\right) + s_2\left(\frac{\nu^2}{3} + \frac{2\nu^4}{15} + \frac{16\nu^6}{63}\right), \quad (|\nu| \ll 1) \tag{2.21}$$

Combining the results obtained (2.20) and (2.21) we obtain the spectral distribution of the pair creation probability in the case when the scattering is weak ($|\nu| \ll 1$)

$$\begin{aligned} \frac{dW_p}{d\varepsilon} &= \frac{dW_p^c}{d\varepsilon} + \frac{dW_p^1}{d\varepsilon} = \frac{\alpha m^2}{2\pi\varepsilon\varepsilon'}\mathrm{Im}\ \left[\Phi_p(\nu) - \frac{1}{2L}F_p(\nu)\right] \\ &= \frac{\alpha m^2}{2\pi\varepsilon\varepsilon'}\frac{2Q}{3}\left[s_1\left(L_1\left(1 - \frac{31\nu_1^4}{21}\right) - \frac{1}{3}\right) + 2s_2\left(L_1\left(1 - \frac{16\nu_1^4}{21}\right) + \frac{1}{6}\right)\right] \\ &= \frac{4Z^2\alpha^3 n_a}{3m^2\omega}\Bigg\{\left(\ln\left(183Z^{-1/3}\right) - f(Z\alpha)\right)\left(1 - \frac{31\nu_1^4}{21}\right) - \frac{1}{6} \\ &+2\frac{\varepsilon^2 + \varepsilon'^2}{\omega^2}\left[\left(\ln\left(183Z^{-1/3}\right) - f(Z\alpha)\right)\left(1 - \frac{16\nu_1^4}{21}\right) + \frac{1}{12}\right]\Bigg\}, \end{aligned} \tag{2.22}$$

where L_1 is defined in (2.3). Integrating (2.22) over ε we obtain

$$W_p = \frac{28Z^2\alpha^3 n_a}{9m^2}\left[\left(\ln(183Z^{-1/3}) - f(Z\alpha)\right)\left(1 - \frac{3312}{2401}\frac{\omega^2}{\omega_0^2}\right) - \frac{1}{42}\right], \tag{2.23}$$

where

$$\omega_0 = m\left(2\pi Z^2\alpha^2 n_a\lambda_c^3 L_1\right)^{-1} \tag{2.24}$$

Note that in gold $\omega_0 = 10.5$ TeV. This is just the value of photon energy starting with the LPM effect becomes essential for the pair creation process in heavy elements. If one omits here the terms $\propto \nu_1^4$ and $\propto (\omega/\omega_0)^2$ these expressions coincide with the known Bethe-Heitler formula for the probability of pair creation by a high-energy photon in the case of complete screening (if one neglects the

contribution of atomic electrons) written down within power accuracy (omitted terms are of the order of powers of $\frac{m}{\omega}$) with the Coulomb corrections, see e.g. Eqs.(19.4) and (19.17) in [14].

The pair creation spectral probability dW/dx vs $x = \varepsilon/\omega$ is shown in Fig.1 for different energies. It is seen that for $\omega = 2.5$ TeV which below ω_0 the difference with the Bethe-Heitler probability is rather small. When $\omega > \omega_0$ there is significant difference with the Bethe-Heitler spectrum increasing with ω growth. In Fig.1 are shown the curves (thin lines 2,3,4) obtained in logarithmic approximation $dW_p^c/d\varepsilon$ (2.10), the first correction to the spectral probability $dW_p^1/d\varepsilon$ (2.15), curves $c2, c3, c4$ and the sum of these two contributions: curves $T1, T2, T3, T4$. It should be noted that for our definition of the parameter ϱ_c (2.6) the corrections are not exceed 6% of the main term. The corrections are maximal for $\nu_0 \sim 3$.

The total probability of pair creation in the logarithmic approximation can be presented as (see (2.10))

$$\frac{W_p^c}{W_{p0}^{BH}} = \frac{9}{14}\frac{\omega_0}{\omega}\mathrm{Im}\int\limits_0^1 \frac{dy}{y(1-y)}\Bigg[\left(\ln p - \psi\left(p+\frac{1}{2}\right)\right)$$
$$+\left(1-2y+2y^2\right)\left(\psi\left(p\right) - \ln p + \frac{1}{2p}\right)\Bigg], \tag{2.25}$$

where

$$p = \frac{bs}{4}, \quad s = \frac{1}{\sqrt{y(1-y)}}, \quad b = \exp\left(i\frac{\pi}{4}\right)\sqrt{\frac{L_1}{L_c}\frac{\omega_0}{\omega}},$$

W_{p0}^{BH} is the Bethe-Heitler probability of pair photoproduction in the logarithmic approximation. The total probability of pair creation W_p^c in gold is given in Fig.2 (curve 2),it reduced by 10% at $\omega \simeq 9$ TeV and it cuts in half at $\omega \simeq 130$ TeV.

At $\nu_0 \gg 1$ the main term of the function $F_p(\nu)$ (see (2.15) and (2.19)) can be written in the form

$$F_p(\nu) = \int\limits_0^\infty \frac{dz}{\sinh^2 z}\left[s_1 f_1(z) - 2is_2 f_2(z)\right]. \tag{2.26}$$

Integrating over z we obtain

$$-\mathrm{Im}\ F_p(\nu) = \frac{\pi}{4}(s_1 - s_2) + \frac{\nu_0}{\sqrt{2}}\left(\ln 2 - C + \frac{\pi}{4}\right)s_2, \tag{2.27}$$

where we take into account the next terms of the decomposition in the term $\propto s_2$. Under the same conditions ($\nu_0 \gg 1$) the function Im $\Phi_p(\nu)$ (2.10) is

$$\mathrm{Im}\ \Phi_p(\nu) = \frac{\pi}{4}(s_1 - s_2) + \frac{\nu_0}{\sqrt{2}}s_2. \tag{2.28}$$

Thus, at $\nu_0 \gg 1$ the relative contribution of the first correction $\dfrac{dW_p^1}{d\varepsilon}$ is defined by

$$r = \frac{dW_p^1}{dW_p^c} = \frac{1}{2L_c}\left(\ln 2 - C + \frac{\pi}{4}\right) \simeq \frac{0.451}{L_c}. \qquad (2.29)$$

In this expression the value r with the accuracy up to terms $\sim 1/L_c^2$ doesn't depend on the energy:$L_c \simeq L_1 + \ln(\omega/\omega_0)/2$. Hence we can find the correction to the total probability at $\omega \gg \omega_0$. The maximal value of the correction is attained at $\omega \sim 10\omega_0$, it is $\sim 6\%$ for heavy elements.

When the parameter ν_0^2 is not very large ($\nu_0 < 10^3,\ \varrho_c > R_n$, see (2.8)) one can solve the equation $\nu_0^2 \varrho_c^4 = 1$ (2.17) using the method of successive approximations. In the first approximation we have

$$\nu_0^2 = \nu_1^2 L_c, \quad L_c \simeq L_1\left(1 + \frac{\ln \nu_1}{L_1}\vartheta(\nu_1 - 1)\right),$$
$$\nu_0 \simeq \frac{1}{\cosh \xi}\sqrt{\frac{\omega}{\omega_0}}\left[1 + \frac{1}{4L_1}\left(\ln\frac{\omega}{\omega_0} - 2\ln\cosh\xi\right)\right]. \qquad (2.30)$$

It should be noted that the relative error in the expression for L_c at $\varrho_c > R_n$ is less than $\ln 2/(4L_1) \leq 2.5\%$. Here we introduce variable ξ

$$\frac{\varepsilon}{\omega} = \frac{1}{2}(1 + \tanh\xi), \quad \frac{\varepsilon\varepsilon'}{\omega^2} = \frac{1}{4\cosh^2\xi}, \quad -\infty < \xi < \infty. \qquad (2.31)$$

Substituting the terms $\propto \nu_0$ in the asymptotic formulas (2.28) and (2.27) into the first line of Eq.(2.22) we obtain expression which contain the integral of the type

$$\int\limits_{-\infty}^{\infty} \frac{d\xi}{\cosh\xi}\left(1 - \frac{1}{2\cosh^2\xi}\right)[A + B\ln\cosh\xi)] = \frac{3\pi}{4}\left[A + B\left(\ln 2 + \frac{1}{6}\right)\right]. \qquad (2.32)$$

Using this result we obtain the total probability of pair creation under strong influence of multiple scattering ($\nu_0 \gg 1$, but not very large)

$$W_p \simeq \frac{3\alpha}{4\sqrt{2}}\frac{m^2}{\sqrt{\omega\omega_0}}\left[1 + \frac{1}{4L_1}\left(\ln\frac{\omega_0}{\omega} + D\right)\right]$$
$$= \frac{3\pi Z^2\alpha^3 n_a L_1}{2\sqrt{2}m^2}\sqrt{\frac{\omega_0}{\omega}}\left[1 + \frac{1}{4L_1}\left(\ln\frac{\omega}{\omega_0} + D\right)\right],$$
$$D = \frac{\pi}{2} - 2C - \frac{1}{3} \simeq 0.08303 \simeq \frac{1}{12.04}. \qquad (2.33)$$

It should be noted that only the main term of the decomposition ($\propto \nu_0$) can be used in Eqs.(2.27) and (2.28) for the calculation of the total probability of pair creation. In the interval $\omega \gg \omega_0$ the contribution into the correction terms

gives also the region where $\cosh^2\xi \sim \omega/\omega_0$, where the parameter $\nu_0 \sim 1$ and the expansion used in Eq.(2.26) is ineligible. The next terms (without corrections $\propto 1/L_1$) are found in Appendix A (Eq.(A.12)), so we have

$$W_p \simeq \frac{3\alpha}{4\sqrt{2}}\frac{m^2}{\sqrt{\omega\omega_0}}\left[1-\frac{\sqrt{2}}{3}(4\ln 2-1)\sqrt{\frac{\omega_0}{\omega}}-\frac{\pi^2}{18}\frac{\omega_0}{\omega}+\frac{1}{4L_1}\left(\ln\frac{\omega}{\omega_0}+D\right)\right]. \tag{2.34}$$

In terms of the Bethe-Heitler total probability of pair creation this result is

$$\frac{W_p}{W_p^{BH}} \simeq 2.14\sqrt{\frac{\omega_0}{\omega}}\left[1-0.836\sqrt{\frac{\omega_0}{\omega}}-0.548\frac{\omega_0}{\omega}+\frac{1}{4L_1}\left(\ln\frac{\omega}{\omega_0}+0.274\right)\right] \tag{2.35}$$

3 Influence of the multiple scattering on the bremsstrahlung

The spectral radiation intensity obtained in [9] (see Eq.(2.39)) has the form

$$dI = \omega dW = \frac{\alpha m^2 x dx}{2\pi(1-x)}\mathrm{Im}\ \left[\Phi(\nu)-\frac{1}{2L_c}F(\nu)\right],\quad x=\frac{\omega}{\varepsilon}, \tag{3.1}$$

where

$$\begin{aligned}
\Phi(\nu) &= \int\limits_0^\infty dz e^{-it}\left[r_1\left(\frac{1}{\sinh z}-\frac{1}{z}\right)-i\nu r_2\left(\frac{1}{\sinh^2 z}-\frac{1}{z^2}\right)\right] \\
&= r_1\left(\ln p-\psi\left(p+\frac{1}{2}\right)\right)+r_2\left(\psi(p)-\ln p+\frac{1}{2p}\right), \\
F(\nu) &= \int\limits_0^\infty \frac{dz e^{-it}}{\sinh^2 z}\left[r_1 f_1(z)-2ir_2 f_2(z)\right], \\
t &= \frac{z}{\nu},\quad r_1 = x^2,\quad r_2 = 1+(1-x)^2.
\end{aligned} \tag{3.2}$$

where $z=\nu t,\ p=i/(2\nu),\ \psi(x)$ is the logarithmic derivative of the gamma function.Some details of the derivation of the second line can be found in Appendix A (see (A.1)-(A.8)). The functions $f_1(z)$ and $f_2(z)$ are defined by Eq.(2.15),

$$\begin{aligned}
&\nu^2 = i\nu_0^2,\quad \nu_0^2 = |\nu|^2 \simeq \nu_1^2\left(1+\frac{\ln\nu_1}{L_1}\vartheta(\nu_1-1)\right),\quad \nu_1^2 = \frac{\varepsilon}{\varepsilon_0}\frac{1-x}{x}, \\
&\varepsilon_0 = m\left(8\pi Z^2\alpha^2 n_a\lambda_c^3 L_1\right)^{-1},\quad L_c \simeq L_1\left(1+\frac{\ln\nu_1}{L_1}\vartheta(\nu_1-1)\right)
\end{aligned} \tag{3.3}$$

Note, that the parameter ε_0 is four times smaller than the parameter ω_0 defined in Eq.(2.24). The LPM effect manifests itself when

$$\nu_1(x_c)=1,\quad x_c=\frac{\varepsilon}{\varepsilon_0+\varepsilon}. \tag{3.4}$$

The formulas derived in [9] and written down above are valid for any energy. In Fig.3 the spectral radiation intensity in gold ($\varepsilon_0 = 2.5$ TeV) is shown for different energies of the initial electron. In the case when $\varepsilon \ll \varepsilon_0$ ($\varepsilon = 25$ GeV and $\varepsilon = 250$ GeV) the LPM suppression is seen in the soft part of the spectrum only for $x \leq x_c \simeq \varepsilon/\varepsilon_0 \ll 1$ while in the region $\varepsilon \geq \varepsilon_0$ ($\varepsilon = 2.5$ TeV and $\varepsilon = 25$ TeV) where $x_c \sim 1$ the LPM effect is significant for any x. For relatively low energies $\varepsilon = 25$ GeV and $\varepsilon = 8$ GeV used in famous SLAC experiment [7], [8] we have analyzed the soft part of spectrum, including all the accompanying effects: the boundary photon emission, the multiphoton radiation and influence of the polarization of the medium. The perfect agreement of the theory and data was achieved in the whole interval of measured photon energies (200 keV$\leq$ $\omega \leq$500 MeV), see the corresponding figures in [9],[10],[11]. It should be pointed out that both the correction term with $F(\nu)$ and the Coulomb corrections have to be taken into account for this agreement.

In the case $\varepsilon \ll \varepsilon_0$ in the hard part of spectrum ($1 \geq x \gg x_c$) the parameter $\nu_1^2 \simeq x_c/x \ll 1$ and the contribution into the integral (3.2) give the region $z \ll 1$. Using the decomposition (C.1) we find (compare with (2.21), (2.22))

$$\begin{aligned}
&\mathrm{Im}\ \Phi(\nu) \simeq r_1\frac{\nu_1^2}{6}\left(1-\frac{31}{21}\nu_1^4\right)+r_2\frac{\nu_1^2}{3}\left(1-\frac{16}{21}\nu_1^4\right), \\
&-\mathrm{Im}\ F(\nu) = -\frac{1}{9}(r_2-r_1)\nu_1^2(1+O(\nu_1^4)). \qquad (3.5)
\end{aligned}$$

In the last formula, which presents corrections $\sim 1/L_1$ we restricted ourselves to the main terms of expansion. Substituting into (3.1) we have

$$\begin{aligned}
&\frac{dI}{dx} = \frac{2Z^2\alpha^3 n_a\varepsilon}{3m^2}\left[r_1\left(L_1\left(1-\frac{31}{21}\frac{x_c^2}{x^2}(1-x)^2\right)-\frac{1}{3}\right)\right. \\
&\left.+2r_2\left(L_1\left(1-\frac{16}{21}\frac{x_c^2}{x^2}(1-x)^2\right)+\frac{1}{6}\right)\right] \qquad (3.6)
\end{aligned}$$

Note that if neglect here the terms $\propto x_c^2/x^2$ we obtain the Bethe-Heitler intensity spectrum with the Coulomb corrections.

In the case $\varepsilon \geq \varepsilon_0$ the intensity spectrum differs from the Bethe-Heitler one at $x \sim 1$ also. When $\varepsilon \gg \varepsilon_0$ one can use the asymptotic expansions (2.27) and (2.28) in the interval not very close to the end of the spectrum ($x = 1$):

$$\begin{aligned}
&\frac{dI}{dx} = \frac{\alpha m^2\nu_0 x}{2\sqrt{2}\pi(1-x)}\left[\frac{r_1\pi}{2\sqrt{2}\nu_0}+r_2\left(1-\frac{\pi}{2\sqrt{2}\nu_0}\right)+r\right] \\
&\simeq \frac{2\sqrt{2}Z^2\alpha^3 n_a\varepsilon}{m^2}\sqrt{\frac{\varepsilon_0 x}{\varepsilon(1-x)}}\left(1+\frac{1}{4L_1}\ln\frac{\varepsilon(1-x)}{\varepsilon_0 x}\right)\left[x^2\right. \\
&\left.+2(1-x)\left(1-\frac{\pi}{2\sqrt{2}}\sqrt{\frac{\varepsilon_0 x}{\varepsilon(1-x)}}\right)+r\right], \quad \varepsilon(1-x) \gg \varepsilon_0 x. \qquad (3.7)
\end{aligned}$$

Now we turn to the integral characteristics of radiation. The total intensity of radiation in the logarithmic approximation can be presented as (see (3.1))

$$\frac{I}{\varepsilon}L_{rad}^0 = 2\frac{\varepsilon_0}{\varepsilon}\mathrm{Im}\Bigg[\int\limits_0^1 \frac{dx}{g}\sqrt{\frac{x}{1-x}}(2(1-x)+x^2)$$
$$+\int\limits_0^1 \frac{x^3dx}{1-x}\left(\psi(p+1)-\psi\left(p+\frac{1}{2}\right)\right)+2\int\limits_0^1 xdx\left(\psi\left(p+1\right)-\ln p\right)\Bigg], \quad (3.8)$$

where

$$p=\frac{g\eta}{2},\quad \eta=\sqrt{\frac{x}{1-x}},\quad g=\exp\left(i\frac{\pi}{4}\right)\sqrt{\frac{L_1}{L_c}\frac{\varepsilon_0}{\epsilon}},$$

L_{rad}^0 is the radiation length in the logarithmic approximation. The relative energy losses of electron per unit time in terms of the Bethe-Heitler radiation length L_{rad}^0: $\frac{I}{\varepsilon}L_{rad}^0$ in gold is given in Fig.2 (curve 1), it reduces by 10% (15% and 25%) at $\varepsilon \simeq 700$ GeV ($\varepsilon \simeq 1.4$ TeV and $\varepsilon \simeq 3.8$ TeV) respectively, and it cuts in half at $\omega \simeq 26$ TeV. This increase of effective radiation length can be important in electromagnetic calorimeters operating in detectors on colliders in TeV range. The contribution of the correction terms was discussed after (2.29). It is valid for the radiation process also.

The spectral distribution of bremsstrahlung intensity and the spectral distribution over energy of created electron (positron) as well as the reduction of energy loss and the photon conversion cross section was calculated by Klein [13], [8] using the Migdal [3] formulas. As was explained above (after Eq.(2.10)) we use more accurate procedure of fine tuning and because of this our calculation in logarithmic approximation differs from Migdal one. We calculated also the correction term and include the Coulomb corrections. For this reason the results shown here in Figs.1-3 are more precise than given in [13], [8].

In Eqs.(3.6) and (3.7) we can use the main terms of decomposition only. The main term in (3.6) gives after the integration over x the standard expression for the radiation length L_{rad} without influence of multiple scattering. The correction term is calculated in Appendix C (see (C.9)) where we need to put $|\beta|^2=\varepsilon_0/\varepsilon \gg 1$

$$\frac{I}{\varepsilon}=\frac{\alpha m^2}{4\pi\varepsilon_0}\left(1+\frac{1}{9L_1}-\frac{4\pi}{15}\frac{\varepsilon}{\varepsilon_0}\right)\simeq L_{rad}^{-1}\left(1-\frac{4\pi}{15}\frac{\varepsilon}{\varepsilon_0}\right),$$
$$\frac{1}{L_{rad}}=\frac{2Z^2\alpha^3n_aL_1}{m^2}\left(1+\frac{1}{9L_1}\right) \quad (3.9)$$

The integration over x of the main term in (3.7) gives (terms $\propto \sqrt{\varepsilon_0/\varepsilon}$ in the square brackets are neglected)

$$I_0 \simeq \frac{9\pi Z^2\alpha^3 n_a\sqrt{\varepsilon\varepsilon_0}}{4\sqrt{2}m^2}L_1\left[1+\frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0}-\frac{46}{27}\right)+r_0\right]$$

$$r_0 = \frac{1}{2L_1}\left(\ln 2 - C + \frac{\pi}{4}\right). \tag{3.10}$$

The corrections (without terms $\propto 1/L_1$) to (3.10) are calculated in Appendix B (see Eq.(B.11)). The complete result is

$$\begin{aligned}
I &= \frac{9\alpha m^2}{32\sqrt{2}}\sqrt{\frac{\varepsilon}{\varepsilon_0}}\left[1 - \frac{4\sqrt{2}}{9}(4\ln 2 + 1)\sqrt{\frac{\varepsilon_0}{\varepsilon}} - \frac{25\pi^2}{54}\frac{\varepsilon_0}{\varepsilon}\right.\\
&\left.+\frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0} + 2\ln 2 - 2C + \frac{\pi}{2} - \frac{46}{27}\right)\right],\\
\frac{I}{\varepsilon L_{rad}} &\simeq \frac{5}{2}\sqrt{\frac{\varepsilon_0}{\varepsilon}}\left[1 - 2.37\sqrt{\frac{\varepsilon_0}{\varepsilon}} - 4.57\frac{\varepsilon_0}{\varepsilon} + \frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0} - 0.3455\right)\right]
\end{aligned} \tag{3.11}$$

Although the coefficients in the last expression are rather large at two first terms of the decomposition over $\sqrt{\varepsilon_0/\varepsilon}$ this formula has the accuracy of the order of 10% at $\varepsilon \sim 10\varepsilon_0$. The integral probability of radiation was calculated in [11]

$$W = \frac{4}{3L_{rad}}\left(\ln\frac{\varepsilon_0}{\varepsilon} + C_2\right), \quad C_2 = 2C - \frac{5}{8} + \int\limits_0^\infty \ln z\left(\frac{1}{z^3}\frac{\cosh z}{\sinh^3 z}\right) \simeq 1.96 \tag{3.12}$$

In the case $\varepsilon \gg \varepsilon_0$ we can calculate the integral probability of radiation starting with Eq.(3.7). Conserving the main term, dividing it by $x\varepsilon$ and integrating over x we find

$$W_0 = \frac{11\pi Z^2\alpha^3 n_a}{2\sqrt{2}m^2}\sqrt{\frac{\varepsilon_0}{\varepsilon}}L_1\left[1 + \frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0} + \frac{8}{11}\right) + r_0\right] \tag{3.13}$$

The correction terms to Eq.(3.12) are calculated in Appendix B (see Eq.(B.13)). Substituting we have

$$\begin{aligned}
W &= \frac{11\alpha m^2}{16\sqrt{2\varepsilon\varepsilon_0}}\left[1 - \frac{4\sqrt{2}}{11}(2\ln 2 + 1)\sqrt{\frac{\varepsilon_0}{\varepsilon}} + \frac{\pi^2}{6}\frac{\varepsilon_0}{\varepsilon}\right.\\
&\left.+\frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0} + 2\ln 2 - 2C + \frac{\pi}{2} + \frac{8}{11}\right)\right]\\
&= \frac{11\pi Z^2\alpha^3 n_a}{2\sqrt{2}m^2}\sqrt{\frac{\varepsilon_0}{\varepsilon}}L_1\left[1 - 1.23\sqrt{\frac{\varepsilon_0}{\varepsilon}} + 1.645\frac{\varepsilon_0}{\varepsilon} + \frac{1}{4L_1}\left(\ln\frac{\varepsilon}{\varepsilon_0} + 2.53\right)\right].
\end{aligned} \tag{3.14}$$

Ratio of the main terms of Eqs.(3.11) and (3.14) gives the mean energy of radiated photon

$$\bar{\omega} = \frac{9}{22}\varepsilon \simeq 0.409\varepsilon. \tag{3.15}$$

A Appendix

We consider the integral which represent the integral probability of pair photoproduction (see Eqs.(2.10) and (2.31))

$$\Pi(a) = \int\limits_0^\infty d\xi \int\limits_0^\infty dz \exp(-az\cosh\xi)\left[\frac{1}{\sinh z} - \frac{1}{z}\right.$$
$$\left. + \frac{1}{a\cosh\xi}\left(1 - \frac{1}{2\cosh^2\xi}\right)\left(\frac{1}{\sinh^2 z} - \frac{1}{z^2}\right)\right]. \tag{A.1}$$

Integrating by parts (over z) the second term of the integrand in (A.1) we have

$$\Pi(a) = -\frac{1}{a}\int\limits_0^\infty \frac{d\xi}{\cosh\xi}\left(1 - \frac{1}{2\cosh^2\xi}\right) + \int\limits_0^\infty d\xi \int\limits_0^\infty dz \exp(-az\cosh\xi)\left[\frac{1}{\sinh z}\right.$$
$$\left. + 1 - \coth z - \frac{1}{2\cosh^2\xi}\left(1 - \coth z + \frac{1}{z}\right)\right]. \tag{A.2}$$

The functions entering in (A.2) we present as

$$\frac{1}{\sinh z} = 2\sum_{k=1}^{\infty} \exp(-(2k-1)z), \quad \coth z - 1 = 2\sum_{k=1}^{\infty} \exp(-2kz) \tag{A.3}$$

Let us consider the integral entering (A.2)

$$\pi_1(a) = \int\limits_0^\infty \frac{d\xi}{\cosh^2\xi} F_1(a\cosh\xi),$$
$$F_1(a\cosh\xi) = \int\limits_0^\infty dz \exp(-az\cosh\xi)\left(1 - \coth z + \frac{1}{z}\right) \tag{A.4}$$

To avoid a divergence of the individual terms in the integral over z we put the lower limit of the integration $\delta \to 0$. Using (A.3) we obtain

$$F_1(x) = \lim_{\delta\to 0}\left[-\mathrm{Ei}(-\delta x) - \sum_{k=1}^{\infty} \frac{\exp(-2k\delta)}{k} + \sum_{k=1}^{\infty} \frac{x}{k(2k+x)}\right] \tag{A.5}$$

Using the expansions

$$-\mathrm{Ei}(-\delta x) = -\ln(\delta x) - C, \quad -\sum_{k=1}^{\infty} \frac{\exp(-2k\delta)}{k} = \ln(1 - \exp(-2\delta)) = \ln 2\delta,$$
$$F_1(x) = \psi\left(\frac{x}{2} + 1\right) - \ln\frac{x}{2}, \tag{A.6}$$

where $\psi(x)$ is the logarithmic derivative of the gamma function, and taking integrals over ξ we have

$$\pi_1(a) = \ln\frac{4}{a} - C - 1 + \frac{a}{4}\sum_{k=1}^{\infty}\frac{1}{k^2} - a^2\sum_{k=1}^{\infty}\frac{1}{k^2\sqrt{4k^2-a^2}}\ln\frac{\sqrt{2k+a}+\sqrt{2k-a}}{\sqrt{2a}} \quad (A.7)$$

The formula (A.2) contains also the integral

$$\pi_2(a) = \int_0^{\infty} d\xi F_2(a\cosh\xi)$$

$$F_2(x) = \int_0^{\infty} dz \exp(-zx)\left(1 - \coth z + \frac{1}{\sinh z}\right) = \psi\left(\frac{x}{2}+1\right) - \psi\left(\frac{x+1}{2}\right) \quad (A.8)$$

Transposing the integration order and using (A.3) we find

$$\pi_2(a) = 2\sum_{k=1}^{\infty}\int_0^{\infty} dz K_0(az)\left(\exp(-(2k-1)z) - \exp(-2kz)\right), \quad (A.9)$$

where $K_0(x)$ is the modified Bessel function. Taking here integrals we have

$$\pi_2(a) = 2\sum_{k=1}^{\infty}\left[\frac{1}{\sqrt{(2k-1)^2-a^2}}\ln\frac{2k-1+\sqrt{(2k-1)^2-a^2}}{a}\right.$$
$$\left. - \frac{1}{\sqrt{4k^2-a^2}}\ln\frac{2k+\sqrt{4k^2-a^2}}{a}\right] \quad (A.10)$$

Substituting (A.7) and (A.10) into Eq.(A.2) we obtain

$$\Pi(a) = -\frac{3\pi}{8a} + \frac{1}{2}\left(\ln\frac{a}{4} + 1 + C\right) - \frac{\pi^3 a}{48}$$
$$+ \sum_{k=1}^{\infty}\left[\frac{a^2}{2k^2\sqrt{4k^2-a^2}}\ln\frac{\sqrt{2k+a}+\sqrt{2k-a}}{\sqrt{2a}}\right.$$
$$+ \frac{2}{\sqrt{(2k-1)^2-a^2}}\ln\frac{2k-1+\sqrt{(2k-1)^2-a^2}}{a}$$
$$\left. - \frac{2}{\sqrt{4k^2-a^2}}\ln\frac{2k+\sqrt{4k^2-a^2}}{a}\right] \quad (A.11)$$

This expression is particularly convenient at $|a| \leq 1$. In the case $|a| \ll 1$ the first three terms of the decomposition are ($a = |a|\exp(i\pi/4)$)

$$\mathrm{Im}\,\Pi(a) \simeq \frac{3\pi}{8\sqrt{2}|a|} + \frac{\pi}{8}(1 - 4\ln 2) - \frac{\pi^3|a|}{48\sqrt{2}} \quad (A.12)$$

B Appendix

Here we consider the asymptotic behavior of the radiation integral characteristics. The integral intensity (the radiation length) can be presented as (see (3.1) and (3.2)):

$$P(\beta) = \int\limits_0^1 \frac{xdx}{1-x}\int\limits_0^\infty dz\exp(-\beta\eta z)\Bigg[x^2\left(\frac{1}{\sinh z}-\frac{1}{z}\right)$$
$$+\frac{1}{\beta\eta}\left(1+(1-x)^2\right)\left(\frac{1}{\sinh^2 z}-\frac{1}{z^2}\right)\Bigg], \quad \eta=\sqrt{\frac{x}{1-x}} \qquad \text{(B.1)}$$

Integrating by parts (over z) the second term of the integrand ($\propto 1/(\beta\eta)$) in (B.1) we have

$$P(\beta) = -\frac{1}{\beta}\int\limits_0^1 \sqrt{\frac{x}{1-x}}\left(1+(1-x)^2\right)dx + \int\limits_0^1 \frac{xdx}{1-x}\int\limits_0^\infty dz\exp(-\beta\eta z)$$
$$\times\Bigg[x^2\left(\frac{1}{\sinh z}-\frac{1}{z}\right)+\left(1+(1-x)^2\right)\left(1-\coth z+\frac{1}{z}\right)\Bigg]$$
$$= -\frac{9\pi}{16\beta}+P_1(\beta)+P_2(\beta), \qquad \text{(B.2)}$$

where

$$P_1(\beta) = \int\limits_0^1 \frac{x^3dx}{1-x}\int\limits_0^\infty dz\exp(-\beta\eta z)\left(1-\coth z+\frac{1}{\sinh z}\right),$$
$$P_2(\beta) = 2\int\limits_0^1 xdx\int\limits_0^\infty dz\exp(-\beta\eta z)\left(1-\coth z+\frac{1}{z}\right), \qquad \text{(B.3)}$$

We split the function $P(\beta)$ into two functions:

$$P_1(\beta) = P_{11}(\beta)+P_{12}(\beta),$$
$$P_{11}(\beta) = \int\limits_0^1 \frac{dx}{1-x}\int\limits_0^\infty dz\exp(-\beta\eta z)\left(1-\coth z+\frac{1}{\sinh z}\right),$$
$$P_{12}(\beta) = -\int\limits_0^1 (1+x+x^2)dx\int\limits_0^\infty dz\exp(-\beta\eta z)\left(1-\coth z+\frac{1}{\sinh z}\right). \qquad \text{(B.4)}$$

Transposing the integration order in $P11(\beta)$ we get the integral over x

$$\int\limits_0^1 \frac{dx}{1-x}\exp(-\beta\eta z) = 2\int\limits_0^\infty \frac{ydy}{\beta^2z^2+y^2}\exp(-y)$$
$$= -2\ln(\beta z)+\int\limits_0^\infty \ln(\beta^2z^2+y^2)\exp(-y)dy \qquad \text{(B.5)}$$

In the limit $|\beta| \ll 1$ we have discarding terms $\sim \beta^2$

$$P_{11}(\beta) = -2\int\limits_0^\infty dz(\ln(\beta z) + C)\left(1 - \coth z + \frac{1}{\sinh z}\right) \tag{B.6}$$

We use Eq.(A.3) in the calculation of the integral over z in $P_{12}(\beta)$

$$\int\limits_0^\infty dz \exp(-\beta\eta z)\left(1 - \coth z + \frac{1}{\sinh z}\right) = 2\sum_{k=1}^{\infty}\left(\frac{1}{2k-1+\beta\eta} - \frac{1}{2k+\beta\eta}\right)$$
$$\simeq 2\ln 2 - \beta\eta\zeta(2). \tag{B.7}$$

Taking into account that $\beta = |\beta|\exp(i\pi/4)$ we have for $\mathrm{Im}P(\beta)$ at $|\beta| \ll 1$

$$\mathrm{Im}P_{11}(\beta) = -\frac{\pi}{2}\int\limits_0^\infty dz\left(1 - \coth z + \frac{1}{\sinh z}\right) = -\pi\ln 2,$$
$$\mathrm{Im}P_{12}(\beta) = \frac{|\beta|\zeta(2)}{\sqrt{2}}\int\limits_0^1 \sqrt{\frac{x}{1-x}}(1+x+x^2)dx = \frac{19\pi}{16\sqrt{2}}|\beta|\zeta(2). \tag{B.8}$$

We will use Eqs.(A.4)-(A.6) in the calculation of the integral over z in $P_2(\beta)$ Eq.(B.3)

$$\int\limits_0^\infty dz \exp(-\beta\eta z)\left(1 - \coth z + \frac{1}{z}\right) = \ln\frac{2}{\beta\eta} - C + \sum_{k=1}^{\infty}\frac{\beta\eta}{k(2k+\beta\eta)}$$
$$\simeq \ln\frac{2}{\beta\eta} - C + \frac{1}{2}\beta\eta\zeta(2) \tag{B.9}$$

As a result we get

$$\mathrm{Im}P_2(\beta) = 2\int\limits_0^1 x dx\left(-\frac{\pi}{4} + \frac{|\beta|}{2\sqrt{2}}\sqrt{\frac{x}{1-x}}\zeta(2)\right) = -\frac{\pi}{4} + \frac{3\pi|\beta|}{8\sqrt{2}}\zeta(2) \tag{B.10}$$

Substituting (B.8) and (B.10) into (B.2) we obtain

$$\mathrm{Im}P(\beta) = \frac{9\pi}{16\sqrt{2}|\beta|} - \pi\ln 2 - \frac{\pi}{4} + \frac{25\pi|\beta|}{16\sqrt{2}}\zeta(2) + O(\beta^2) \tag{B.11}$$

In the calculation of the total probability of radiation one have to make the substitution in (B.1)

$$\int\limits_0^1 \frac{xdx}{1-x}\ldots \quad \rightarrow \quad \int\limits_0^1 \frac{dx}{1-x}\ldots.$$

Then

$$T(\beta) = -\frac{1}{\beta}\int\limits_0^1 \frac{dx}{\sqrt{x(1-x)}}(1+(1-x)^2) + t_1(\beta) + t_2(\beta), \tag{B.12}$$

where

$$\begin{aligned}
\text{Im } t_1(\beta) &= -\pi\ln 2 + \frac{1}{\sqrt{2}}|\beta|\zeta(2)\int\limits_0^1 \sqrt{\frac{x}{1-x}}(1+x)dx = -\pi\ln 2 + \frac{7\pi}{8\sqrt{2}}|\beta|\zeta(2), \\
\text{Im } t_2(\beta) &= -\frac{\pi}{2} + \frac{1}{2\sqrt{2}}|\beta|\zeta(2)\int\limits_0^1 \sqrt{\frac{x}{1-x}}dx = -\frac{\pi}{2} + \frac{\pi}{2\sqrt{2}}|\beta|\zeta(2), \\
\text{Im } T(\beta) &= \frac{11\pi}{8\sqrt{2}|\beta|} - \frac{\pi}{2} - \pi\ln 2 + \frac{11\pi}{8\sqrt{2}}|\beta|\zeta(2) + O(\beta^2)
\end{aligned} \tag{B.13}$$

C Appendix

Here we consider the asymptotic behavior in the region where the LPM effect is weak. In this region in Eq.(A.1) the parameter $|a| \gg 1$ and in the integral over z the interval $z \ll 1$ contributes, and we can use expansions

$$\begin{aligned}
\frac{1}{\sinh z} - \frac{1}{z} &= -\frac{z}{6} + \frac{7z^3}{360} - \frac{31z^5}{15120} + \dots, \\
\frac{1}{\sinh^2 z} - \frac{1}{z^2} &= -\frac{1}{3} + \frac{z^2}{15} - \frac{2z^4}{189} + \dots
\end{aligned} \tag{C.1}$$

Substituting these expansions into (A.1) and taking into account that $a = |a|\exp(i\pi/4)$ we get

$$\begin{aligned}
\text{Im } \Pi(a) = \int\limits_0^\infty d\xi\Bigg[&\frac{1}{6|a|^2\cosh^2\xi} - \frac{31}{126|a|^6\cosh^6\xi} + \left(1 - \frac{1}{2\cosh^2\xi}\right) \\
&\times\left(\frac{1}{3|a|^2\cosh^2\xi} - \frac{16}{63|a|^6\cosh^6\xi}\right)\Bigg] = \frac{7}{18|a|^2}\left(1 - \frac{184}{343|a|^4}\right).
\end{aligned} \tag{C.2}$$

We turn to Im $T(\beta)$ Eq.(B.1) at $|\beta| \gg 1 (\beta = |\beta|\exp(i\pi/4))$. The integral over z coincides with this integral in (C.1). The integral over x gives for $x \sim 1$ the same structure as in (C.2): the main term $\sim 1/|\beta|^2$ and the correction $\sim 1/|\beta|^6$. In the region $x \sim 1/|\beta|^2$ where the influence of the LPM effect is significant, the correction is proportional to the phase space: $\int xdx \sim 1/|\beta|^2$. This is the main correction. Because of this we split the integration interval over x into two intervals: 1) $0 \le x \le x_0$ and 2) $x_0 \le x \le 1$, where $1/|\beta|^2 \ll x_0 \ll 1$. In the first interval

$$P_1(\beta, x_0) \simeq \frac{2}{\beta}\int\limits_0^{x_0} \frac{xdx}{\eta(1-x)}\int\limits_0^\infty \exp(-\beta\eta z)\left(\frac{1}{\sinh^2 z} - \frac{1}{z^2}\right)dz. \tag{C.3}$$

Integrating over z by part and then over x we have

$$P_1(\beta, x_0) \simeq \frac{2x_0}{3\beta^2} + \frac{8}{\beta^4}\int\limits_0^\infty \frac{dz}{z^2}\left(1 - (\beta z\sqrt{x_0} + 1)\exp(-\beta z\sqrt{x_0})\right)\left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3}\right). \tag{C.4}$$

In the integral which contains the term $\beta z\sqrt{x_0}$ the interval $z \ll 1$ contributes so that

$$\int\limits_0^\infty \frac{dz}{z^2}\beta z\sqrt{x_0}\exp(-\beta z\sqrt{x_0})\left(-\frac{z}{15}\right) = \frac{1}{15} \tag{C.5}$$

In the remaining integral we split the interval of integration into two: 1) $0 \le z \le z_0$, 2)$z_0 \le z < \infty$ $(1/(\beta\sqrt{x_0}) \ll z_0 \ll 1)$, then

$$\begin{aligned}
&\int\limits_0^\infty \frac{dz}{z^2}\left(1 - \exp(-\beta z\sqrt{x_0})\right)\left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3}\right) \\
&\simeq \int\limits_0^{z_0} \frac{dz}{z^2}\left(1 - \exp(-\beta z\sqrt{x_0})\right)\left(-\frac{z}{15}\right) + \int\limits_{z_0}^\infty \frac{dz}{z^2}\left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3}\right) \\
&\simeq -\frac{1}{15}\left(\ln(-\beta z_0\sqrt{x_0}) + C\right) + \int\limits_{z_0}^\infty \frac{dz}{z^2}\left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3}\right)
\end{aligned} \tag{C.6}$$

So we have

$$\text{Im } P_1(\beta, x_0) \simeq \frac{2x_0}{3|\beta|^2} - \frac{2\pi}{15|\beta|^4} \tag{C.7}$$

In the second interval over x $(1 \ge x \ge x_0)$ the interval $z \ll 1$ contributes as well as in the first term of $P(\beta)$ Eq.(B.1) which we include here

$$\text{Im } P_2(\beta, x_0) \simeq \frac{1}{2|\beta|^2} - \frac{2x_0}{3|\beta|^2} + O(\frac{1}{\beta^6}) \tag{C.8}$$

Adding (C.7) and (C.8) we get for $|\beta| \gg 1$

$$\text{Im } P(\beta) \simeq \frac{1}{2|\beta|^2} - \frac{2\pi}{15|\beta|^4} \tag{C.9}$$

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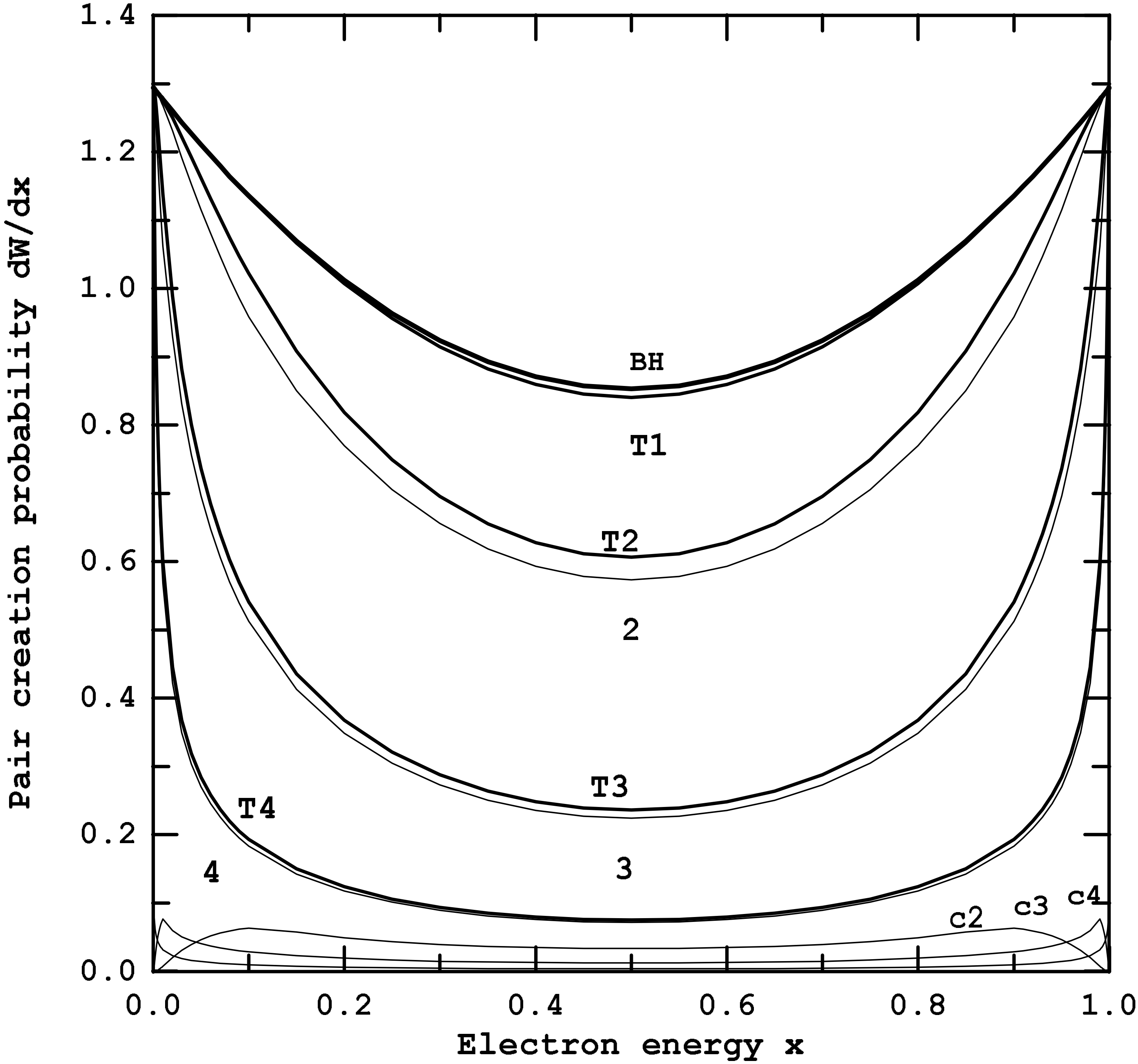
Figure captions

- **Fig.1** The pair creation spectral probability $\frac{dW_p}{dx}, x = \frac{\varepsilon}{\omega}$ in gold in terms of the exact total Bethe-Heitler probability taken with the Coulomb corrections (see Eq.(2.24)).
  - Curve BH is the Bethe-Heitler spectral probability (see Eq.2.23);
  - curve T1 is the total contribution (the sum of the logarithmic approximation $dW_p^c/d\varepsilon$ (2.10) and the first correction to the spectral probability $dW_p^1/d\varepsilon$ (2.15)) for the photon energy $\omega = 2.5$ TeV;
  - curve 2 is the is the logarithmic approximation $dW_p^c/d\varepsilon$ (2.10), curve c2 is the first correction to the spectral probability $dW_p^1/d\varepsilon$ (2.15)) and curve T2 is the sum of the previous contributions for the photon energy $\omega = 25$ TeV;
  - curves 3, c3, T3 are the same for the photon energy $\omega = 250$ TeV;
  - curves 4, c4, T4 are the same for the photon energy $\omega = 2500$ TeV;
- **Fig.2** The relative energy losses of electron per unit time in terms of the Bethe-Heitler radiation length L_{rad}^0: $\frac{I}{\varepsilon}L_{rad}^0$ in gold vs the initial energy of electron (curve 1) and the total pair creation probability per unit time W_p^c (see Eq.(2.25))in terms of the Bethe-Heitler total probability of pair creation W_{p0}^{BH} (see Eq.(2.24)) in gold vs the initial energy of photon (curve 2).
- **Fig.3** The spectral intensity of radiation $\omega\frac{dW}{d\omega} = x\frac{dW}{dx},\ x = \frac{\omega}{\varepsilon}$ in gold in terms of $3L_{rad}$ taken with the Coulomb corrections (see Eq.(3.9)).
  - Curve BH is the Bethe-Heitler spectral intensity (see Eq.3.6);
  - curve 1 is the is the logarithmic approximation $\omega dW_c/d\omega$ Eq.(2.28) of [9], curve c1 is the first correction to the spectral intensity $\omega dW_1/d\omega$ Eq.(2.33) of [9] and curve T1 is the sum of the previous contributions for the electron energy $\varepsilon = 25$ GeV;
  - curve 2 is the is the logarithmic approximation $\omega dW_c/d\omega$ Eq.(2.28) of [9], curve c2 is the first correction to the spectral intensity $\omega dW_1/d\omega$ Eq.(2.33) of [9] and curve T2 is the sum of the previous contributions for the electron energy $\varepsilon = 250$ GeV;
  - curves 3, c3, T3 are the same for the electron energy $\varepsilon = 2.5$ TeV;
  - curves 4, c4, T4 are the same for the electron energy $\varepsilon = 25$ TeV;



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Pair creation probability dW/dx
BH
T1
T2
2
T3
T4
3
4
c2
c3
c4
0.0
0.2
0.4
0.6
0.8
1.0
Electron energy x

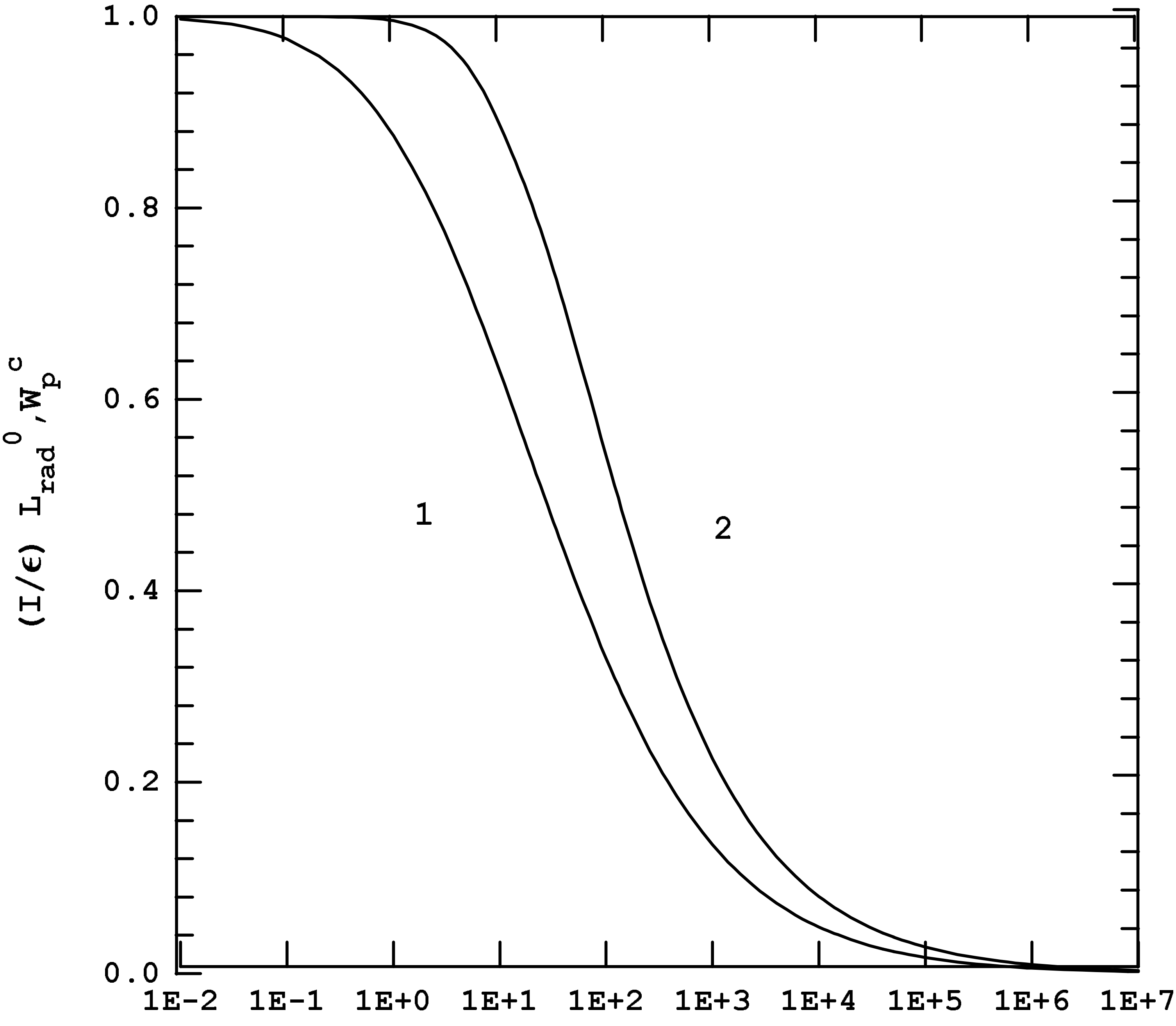
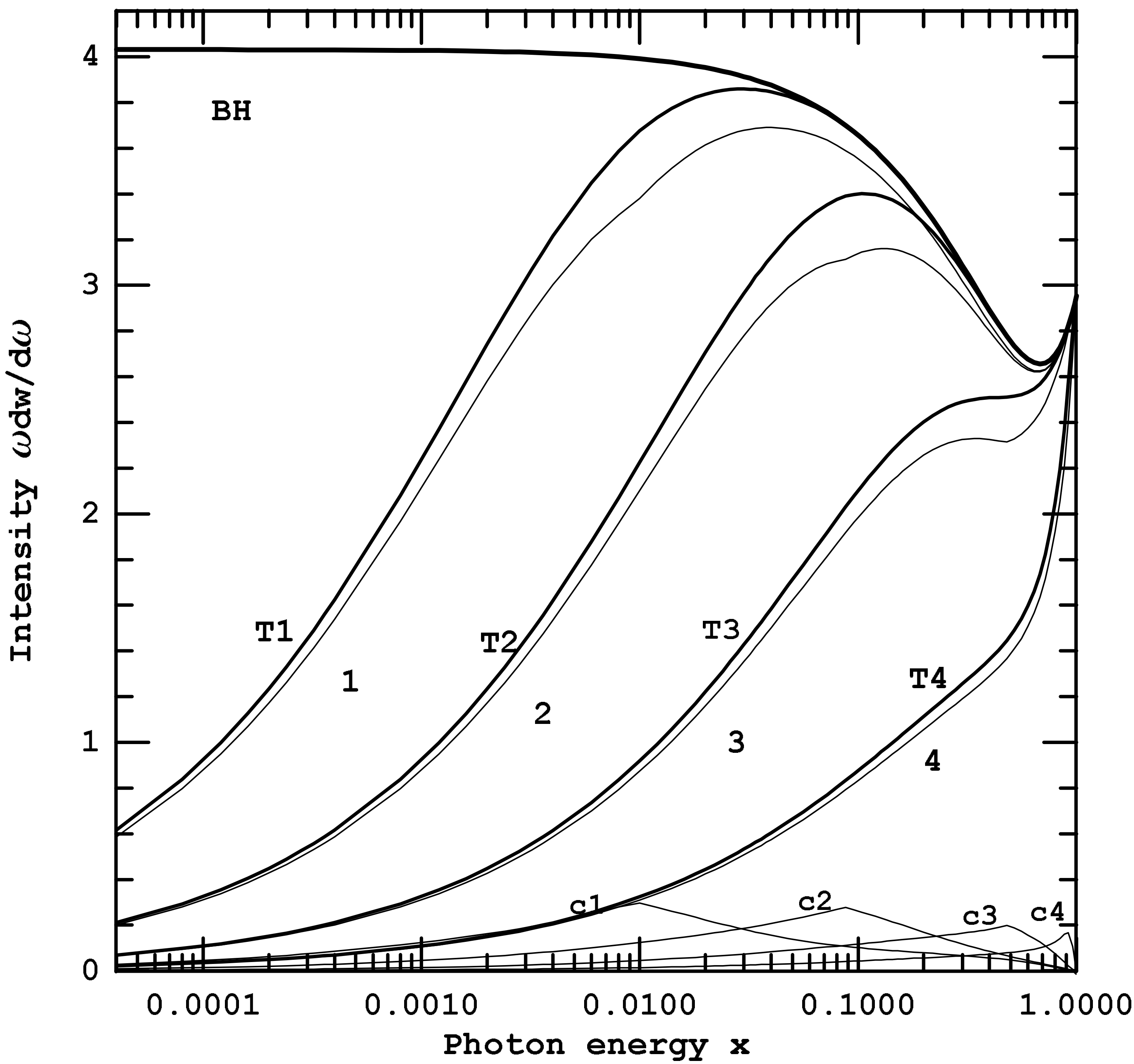

1.0
0.8
0.6
0.4
0.2
0.0
1E-2
1E-1
1E+0
1E+1
1E+2
1E+3
1E+4
1E+5
1E+6
1E+7
Photon energy (TeV)
$(I/\epsilon)\ L_{rad}^{0}, W_p^{c}$
1
2

Fig.2



BH
T1
1
T2
2
T3
3
T4
4
c1
c2
c3
c4
4
3
2
1
0
0.0001
0.0010
0.0100
0.1000
1.0000
Photon energy x
Intensity ωdw/dω